Fractions: Where are the Cookies?
Grades 4 – 6
Formative Assessment Lesson
Problem Solving

Designed and revised by the Kentucky Department of Education
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues with this file, please contact the KDE math team at:
kdemath@education.ky.gov

Revised 2016

Where are the Cookies?

Mathematical goals

This lesson is intended to help you assess how well students are able to use fractions in a problem solving context. In particular this lesson aims to identify and help students with difficulties:

- Conceptualizing fractional parts of different wholes
- Choosing an appropriate, systematic way to collect and organize to display multiple step tasks
- Conceptualizing fractional comparisons and fractional parts of other fractions

Kentucky Academic Standards

This lesson involves a range of *mathematical practices,* with emphasis on:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
7. Look for and make use of structure.

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

> 4.NF **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
> 5.NF **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**
> 6.NS **Apply and extend previous understandings of multiplication and division.**

This lesson unit is structured in the following way:

- Before the lesson, students attempt the task individually. You then review their work and formulate questions for students to answer in order for them to improve their work.
- Students use whiteboards to review fraction task similar to cookie task.
- In the lesson they work collaboratively, in small groups, to critique examples of other students' work.
- In a whole-class discussion, students explain and compare the alternative approaches they have seen and used.
- Finally, students work alone again to improve their individual solutions.

Materials required

- Each individual student will need two copies of the handout *Where are the Cookies?*
- Each student needs a whiteboard & dry erase marker for the short intro lesson.
- Each small group of students will need d a copy of *Sample Responses to Discuss* and whichever samples of student work chosen.

Time needed

Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson. Give each student a copy of *Where are the cookies?* Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering the question. Show your work. Don't worry if you can't figure it out. There will be a lesson on this material [tomorrow] that will help you improve your work. Your goal is to be able to answer this question with confidence by the end of that lesson.


Where are the cookies?

Mrs. James left a tray of cookies on the counter early one morning. Larry walked by before lunch and decided to take 1/3 of the cookies on the tray. Later that afternoon Barry came in and ate 1/4 of the remaining cookies. After supper Terry saw the tray of cookies and ate 1/3 of the cookies remaining at that time. The next morning Mrs. James found the tray with only 6 cookies left. How many cookies were on the tray when Mrs. James first left it on the counter?

It is important that students answer the question without assistance, as far as possible. Students who sit together often produce similar answers, and then, when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their different problem solving approaches. We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions.

Some suggestions for feedback questions based on common misconceptions are given in the chart below. We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help the majority of students. These can be written on the board at the end of the lesson.

Common Issues	Suggested questions and prompts
Student who has trouble getting started.	• *How might you work backwards to begin this task?*
Student does not adjust for a new whole when determining the new amount eaten.	• *How can you make sure you are only getting a fraction of what is left and not the original amount of cookies?*
Student confuses amount eaten with amount left.	• *If 1/3 are eaten what fraction are left?*
Student work is unsystematic.	• *What is the same and what is different after each person eats some cookies?* • *How can you organize your work?*
Student uses the wrong fraction operations.	• *In this problem are we multiplying or adding, dividing or subtracting? How do you know?*

Student writes answer without explanation.	• *How could you explain/show how you reached your conclusions so that someone in another class understands?* • *How can you use numbers, words, or diagrams to describe how the cookies were eaten?*
Student correctly identifies when the number of cookies that started on the tray.	• *Think of another way of solving the problem. Is this method better or worse than your original one? Explain your answer.* • *Can you make a new problem with a different number of cookies left?*

Suggested lesson outline

Whole Class Introduction (10 minutes)

Using whiteboards, have students respond to the following:

We are going to do some more work with solving problems with fractions. Model this problem using at least two different strategies.

Sally has 3 apples and eats ½ of them. Jim eats 1/3 of what Sally had left. How much of the apples are now left?

Have students share responses to this task and explain their pictures or numbers in their solutions. Sample response:  Sally ate 1 ½ of the apples, so there were 1 ½ left when she was finished. Jim ate 1/3 of the 1 ½ that were left  so there were two halves or 1 whole apple left when he was finished.

Be sure to showcase visual representations that varied in strategies, including bar models, visual drawings, expressions, etc.

Collaborative activity: (20 minutes)
Return the students' work on the *Where are the Cookies?* problem. Ask students to re-read both the *Where are the Cookies?* problem and their solutions. Have students share their work with a partner, and have each person ask any clarifying questions to understand their partner's approach to the task.

Make a note of student approaches to the task Give each small group of students a copy of the *Sample Responses to Discuss* handout. Choose the samples of student work that match your students' level of understanding. Begin with a couple of ***incorrect or incomplete*** work samples (Robin, Britney, Charlie & Dawn) and a couple of ***correct*** samples (Katrina, Jim, Eddie & Andrew) depending on the methods and misconceptions held by each group. The student work sample by ***Brandon uses equations*** and should be used as an ***extension*** only for groups who understand the other samples first.

Display the following questions using the provided sheet: *Sample Responses to Discuss*.

Describe the problem solving approach the student used.
For example, you might:

• Describe the way the student has organized the solution.
• Describe what the student did to calculate the number of cookies starting on the tray.
Explain what the student needs to do to complete or correct his or her solution.

This analysis task will give students an opportunity to evaluate a variety of alternative approaches to the task. During small-group work, support student thinking as before. Also, check to see which of the explanations students find more difficult to understand. Identify one or two of these approaches to discuss in the plenary discussion. Note similarities and differences between the sample approaches.

Whole-class discussion comparing different approaches (20 minutes)

Organize a whole-class discussion to consider different approaches to the task. The intention is for you to focus on getting students to understand the methods of working out the answers, rather than either numerical or pictorial solutions. Focus your discussion on the tasks students found difficult.

Let's stop and talk about different approaches.

Ask the students to compare the different solution methods. Begin with the most pictorial and work through to the more abstract approaches – make connections between approaches throughout the discussion. Questions you might pose to students during the discussion:

Which approach did you like best of the student work you analyzed? Why?

Which approach did you find it most difficult to understand?

Read through your original responses and think about what you have learned this lesson.

Did anyone use a different method than the samples of student work you analyzed?

Improving individual solutions to the assessment task (10 minutes)

If you are running out of time, you could schedule this activity to start the next day.

Make sure students have their original individual work on the *Where are the cookies?* task on hand. Give them a fresh, blank copy of the *Where are the cookies?* task sheet.
If a student is satisfied with his or her solution, ask the student to try a different approach to the problem and to compare the approach already used.

Solution & Discussion of Student work samples

The solution to the task is 24 cookies started on the tray. Students could work backwards to solve the task using numbers and/or pictures but they have to keep track of the amount eaten and the amount left after each person eats some of the cookies. It is possible a student might just guess & check to keep trying numbers to begin on the tray and work through the fractions until they find an answer that results in 6 remaining cookies. Using multiplication of fractions and equations are more sophisticated methods, but are not required to solve this task. They are however, efficient approaches that some students will be ready to investigate.

The methods used by Katrina, Dawn, Jim, Eddie, Andrew & Brandon all lead to the **correct solution**. However, Dawn's work is incomplete because it does not give the final answer of 24. Dawn's pictorial model uses the tray as the one whole and subtracts amounts while at the same time showing the fractional parts of the whole. In the end there would be 12 equal size sections and 2 cookies could fit in each so the approach could lead to correct solution, it is just incomplete. Katrina & Jim use similar pictorial methods by actually drawing out cookies by working backwards. Eddie

uses a bar model to work backwards. Andrew uses fraction multiplication and Brandon solves with equations.

Robin, Britney & Charlie all use incorrect methods. Robin attempts to use pictures, Britney uses division & Charlie confuses the amount eaten with the amount left.

This lesson format was designed from the Classroom Challenge Lessons intended for students in grades 6 through 12 from the Math Assessment Project.

Name _____

Where are the cookies?

Mrs. James left a tray of cookies on the counter early one morning. Larry walked by before lunch and decided to take 1/3 of the cookies on the tray. Later that afternoon Barry came in and ate 1/4 of the remaining cookies. After supper Terry saw the tray of cookies and ate 1/2 of the cookies remaining at that time. The next morning Mrs. James found the tray with only 6 cookies left. How many cookies were on the tray when Mrs. James first left it on the counter?

Student Materials

Sample Responses to Discuss

Here is some work on ***Where are the Cookies?*** from students in another class.

For each piece of work:

1. Write the name of the student whose solution you are analyzing.
2. Describe the problem solving approach the student used.

   For example, you might:

   - Describe the way the student has organized the solution.
   - Describe what the student did to calculate the number of cookies that started on the tray.

3. Explain what the student needs to do to complete or correct his or her solution.

_____'s Solution

_____

_____

_____

_____

_____

_____

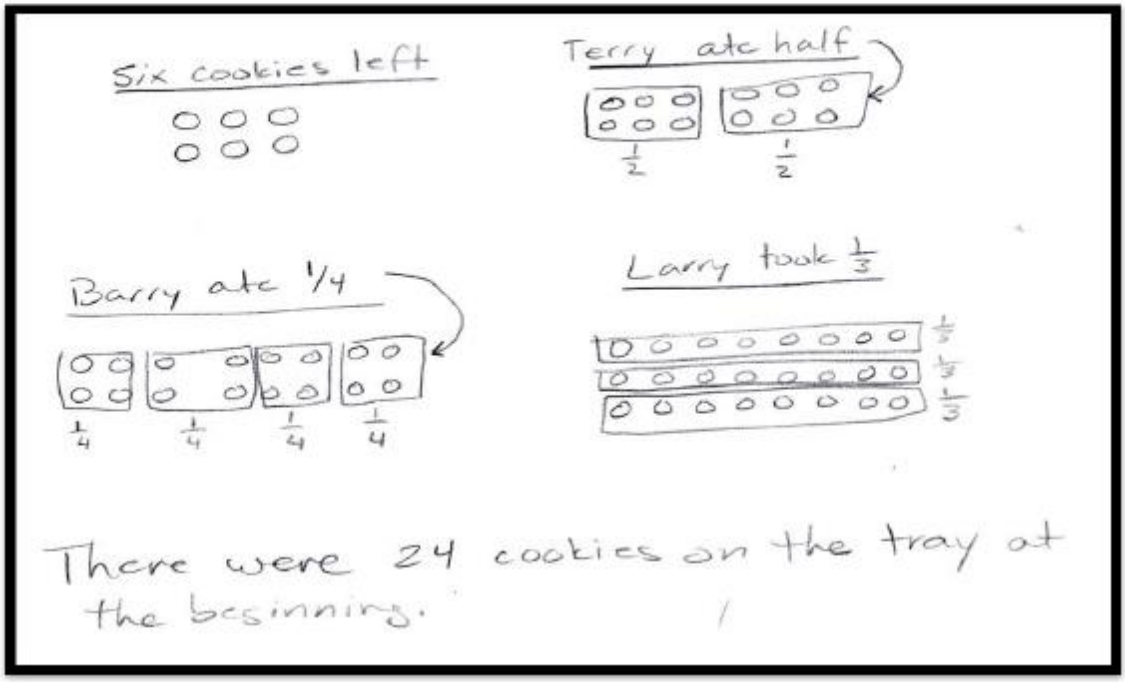
_____

_____

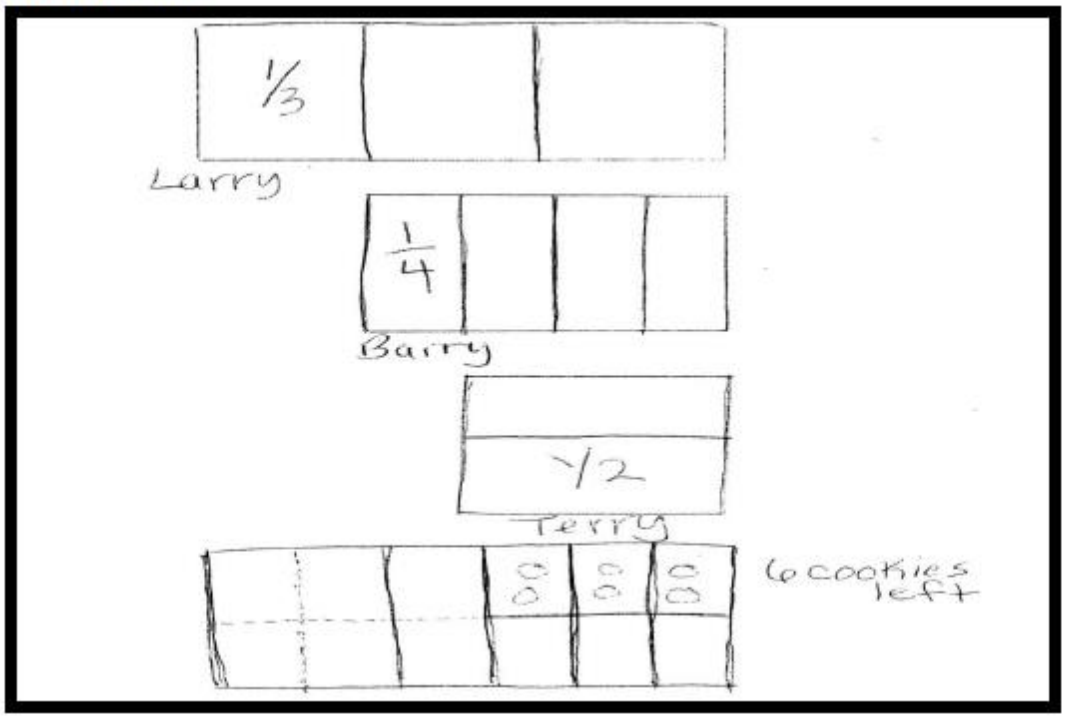
_____'s Solution

_____'s Solution

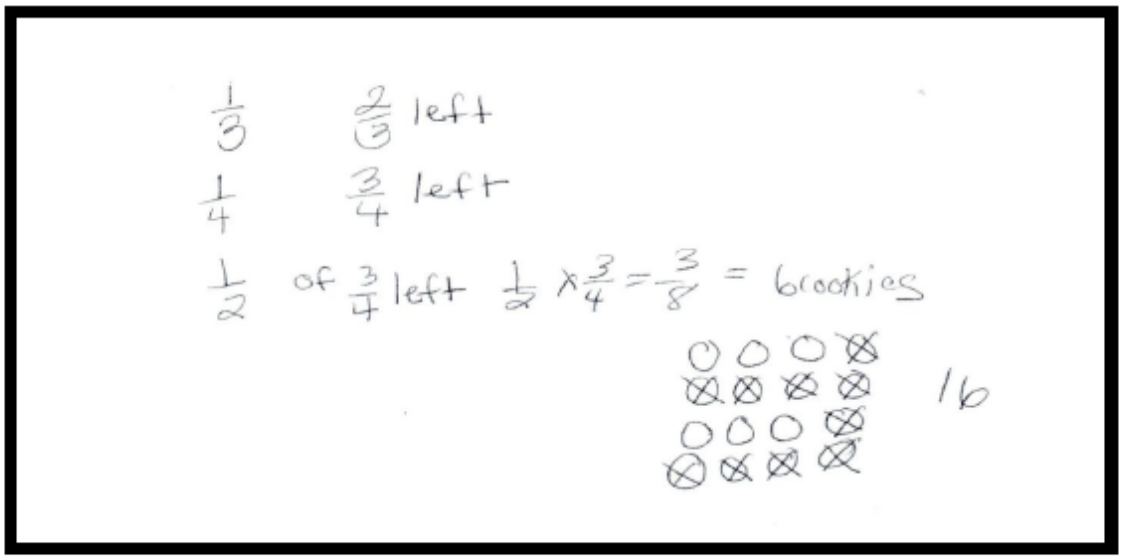
Katrina's Solution



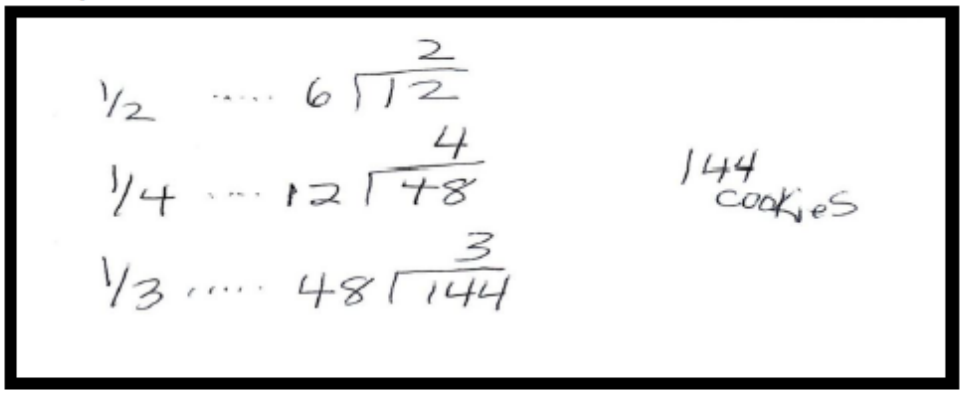
Dawn's Solution



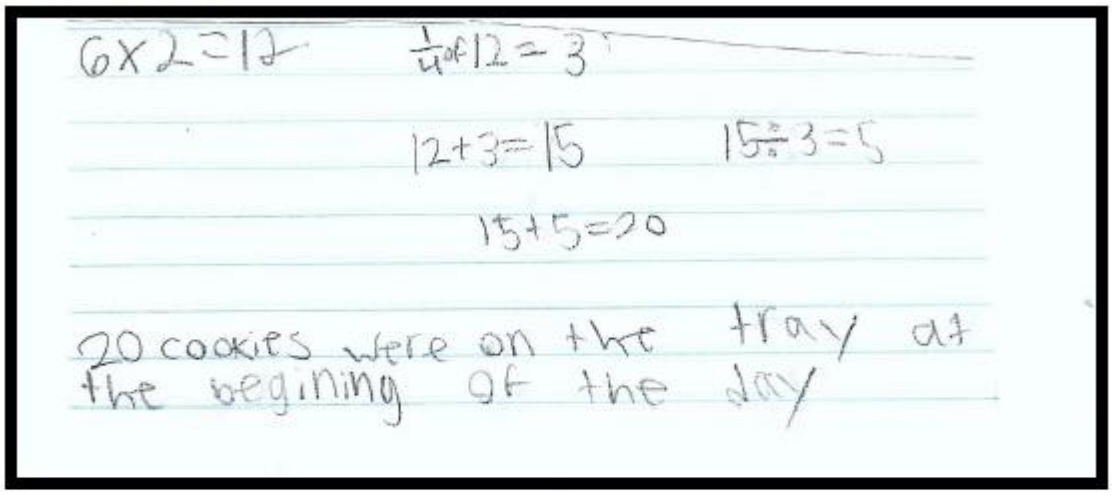
Robin's Solution



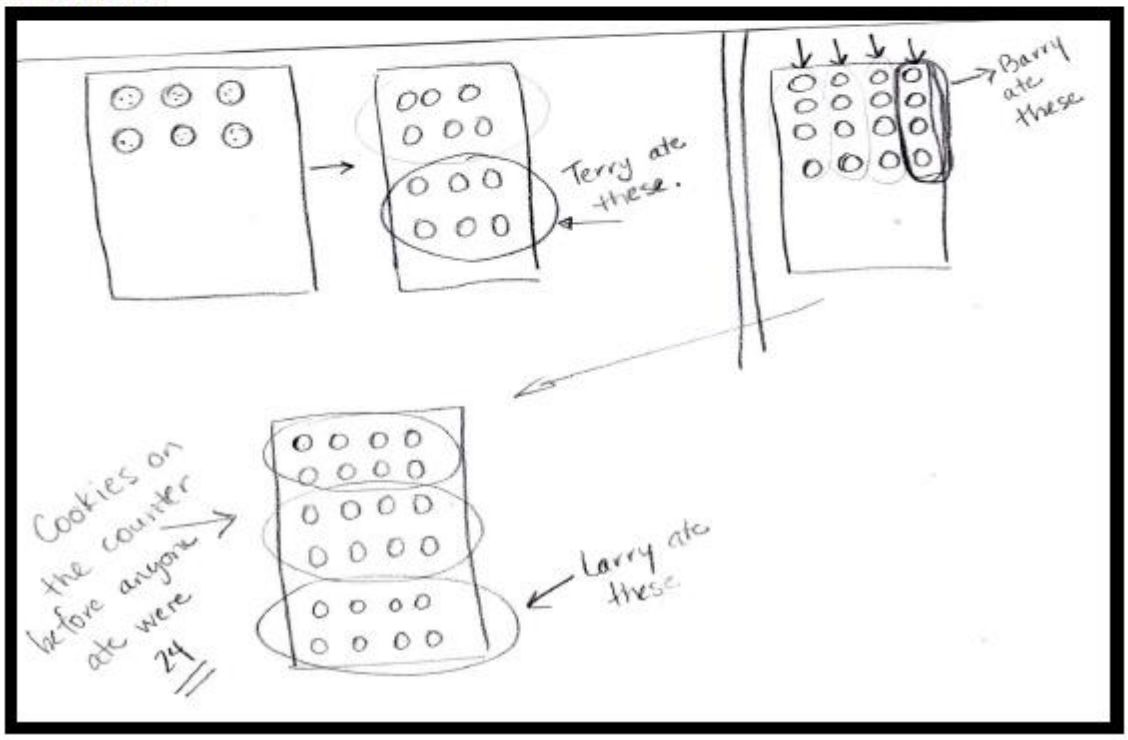
Britney's Solution



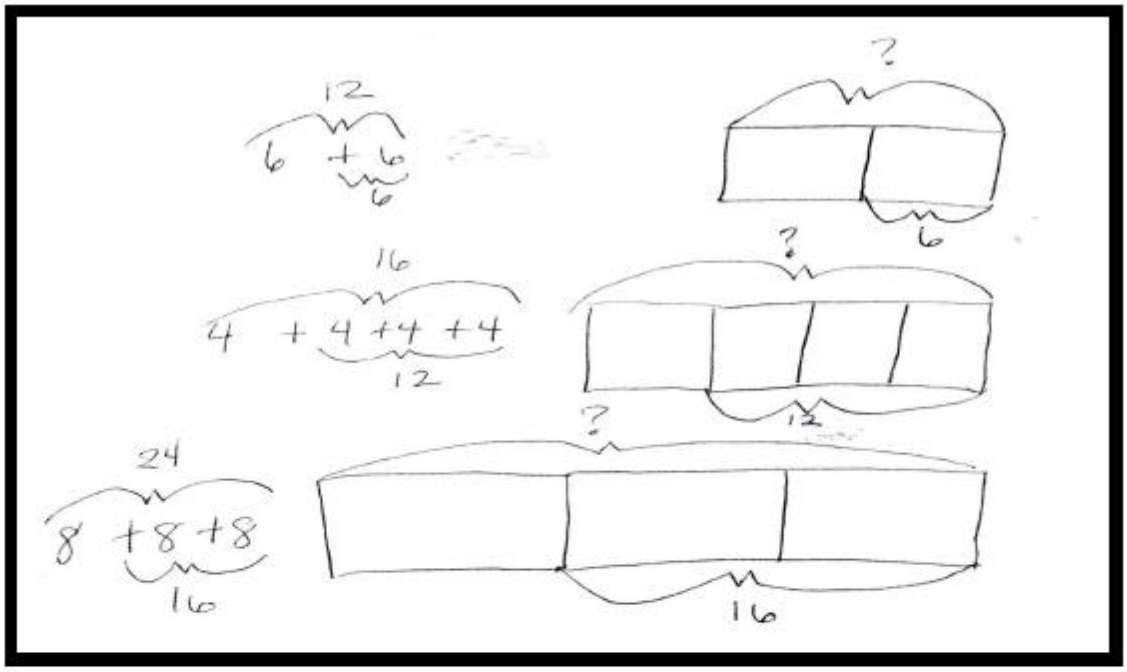
Charlie's Solution



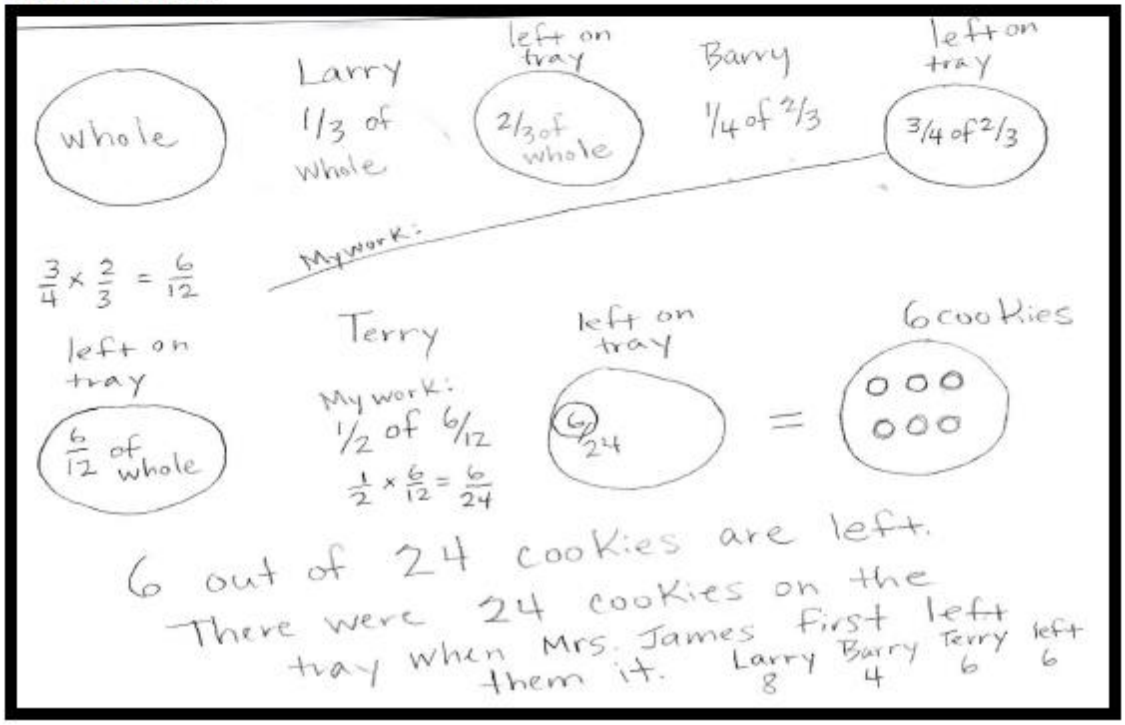
Jim's Solution



Eddie's Solution



Andrew's Solution



Brandon's Solution